Resonance RLC circuit data analysis

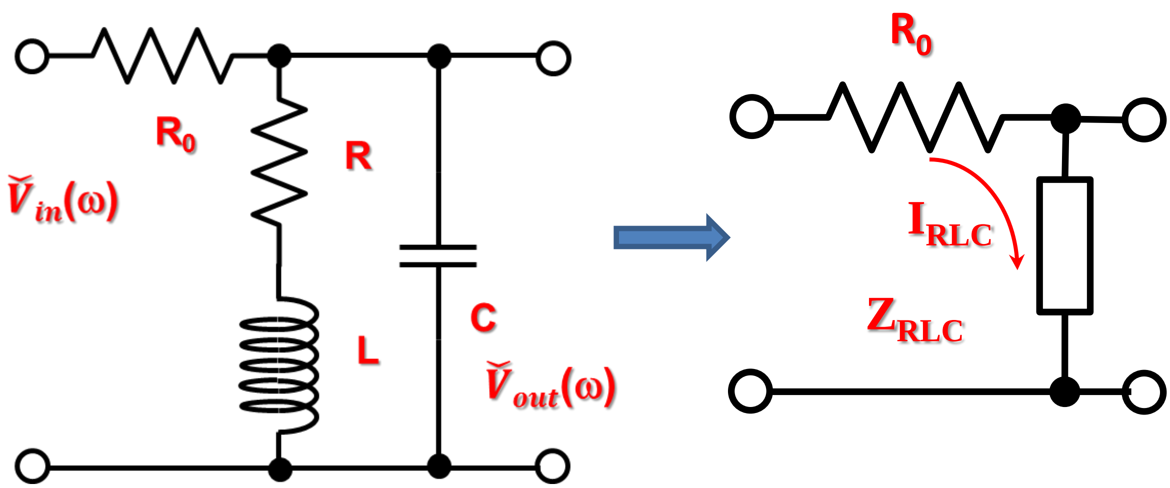

Fig.1. Actual circuit diagram and its equivalent circuit. Here $\breve{Z}_{RLC}$ is the complex impedance of the RLC resonance circuit .

We assuming that $R_0 >> |\breve{Z}_{RLC}|$ and current $\breve{I}_{RLC}$ through $\breve{Z}_{RLC}$ is defined by the R_0 **as** $\breve{I}_{RLC} = \breve{V}_{in}/R_0$**.** The voltage drop on resonance RLC circuit in this case can be written as:

$$\breve{V}_{out} = \breve{Z}_{RLC} \bullet \breve{I}_{RLC} = \frac{\breve{V}_{in}}{R_0} \bullet \breve{Z}_{RLC} \quad \textbf{(1)}$$

The equation for transfer function $\breve{H}(\omega)$ is:

$$\breve{H}(\omega) = \frac{\breve{V}_{out}}{\breve{V}_{in}} = \frac{1}{R_0} \bullet \breve{Z}_{RLC} \quad \textbf{(2)}$$

Finally the equations for $\breve{Z}_{RLC}$ and $\breve{H}(\omega)$ are:

$$\breve{Z}_{RLC} = \frac{R + j\omega L}{1 - \omega^2 LC + j\omega RC} \quad \textbf{(3)}$$

$$\tilde{H}(\omega) \cong \left(\frac{1}{R_0}\right) \frac{R + j\omega L}{1 - \omega^2 LC + j\omega RC} \quad \textbf{(4)}$$

After introducing $\omega_o = 2\pi f = \frac{1}{\sqrt{LC}}$ **(undamped resonance frequency) and** $Q = \frac{1}{\omega_0 RC} = \frac{1}{R}\sqrt{\frac{L}{C}}$ **(quality factor) the equation for transfer function can be rewritten as:**

$$\tilde{H}(\omega) = \left(\frac{R}{R_0}\right) \frac{1 + j\frac{\omega}{\omega_0}Q}{\left(1 - \frac{\omega^2}{\omega_0^2}\right) + j\frac{\omega}{\omega_0}\frac{1}{Q}} = \left(\frac{R}{R_0}\right) \frac{1 - j\frac{\omega}{\omega_0}\left(\frac{1}{Q} - Q\left(1 - \frac{\omega^2}{\omega_0^2}\right)\right)}{\left(1 - \frac{\omega^2}{\omega_0^2}\right)^2 + \left(\frac{\omega}{\omega_0}\right)^2 \frac{1}{Q^2}} \quad \textbf{(5)}$$

Now to simplify the equation we can introduce the reduced frequency $\gamma = \frac{\omega}{\omega_0}$ and transfer function and its both real and imaginary components can presented as:

$$\tilde{H}(\omega) = \left(\frac{R}{R_0} \bullet Q\right) \frac{Q - j\gamma\left(1 - Q^2\left(1-\gamma^2\right)\right)}{Q^2\left(1-\gamma^2\right)^2 + (\gamma)^2}; \qquad (6)$$

$$H_{RE} = \left(\frac{R}{R_0} \bullet Q\right) \frac{Q}{Q^2\left(1-\gamma^2\right)^2 + (\gamma)^2}; \qquad (7)$$

$$H_{IM} = \left(\frac{R}{R_0} \bullet Q\right) \frac{-j\gamma\left(1 - Q^2\left(1-\gamma^2\right)\right)}{Q^2\left(1-\gamma^2\right)^2 + (\gamma)^2} \qquad (8)$$

Equations (7) and (8) can be used as the fitting function for data analysis. Corresponding equations for modulus and phase angle can de derived from (7) and (8) as:

$$\left|\tilde{H}\right| = \left(\frac{R}{R_0} \bullet Q\right) \frac{\sqrt{Q^2 +^2 \left(1 - Q^2\left(1-\gamma^2\right)\right)^2}}{Q^2\left(1-\gamma^2\right)^2 + (\gamma)^2}; \qquad (9)$$

$$\theta = \arctan\left(\frac{H_{IM}}{H_{RE}}\right) = \arctan\left(\frac{-\gamma\left(1 - Q^2\left(1-\gamma^2\right)\right)}{Q}\right) \qquad (10)$$

Now let us use the equations (7)-(10) for writing the fitting functions in Origin do the fitting to the experimental data. As an example of how it should be done I used the data obtained by ***Tsung-Lin Hsieh*** in Fall 2011.

To avoid the correlations between fitting parameters we can use only three of those and it will be **Q,** scaling coefficient $\left(\frac{R}{R_0} Q\right)$ and resonance frequency ω_0 (or f_0). For fitting you have to create the graph in Origin and go to Analysis menu: **Analysis → Fitting → Nonlinear Curve Fit → Open Dialog…**

1. Real part (X) analysis

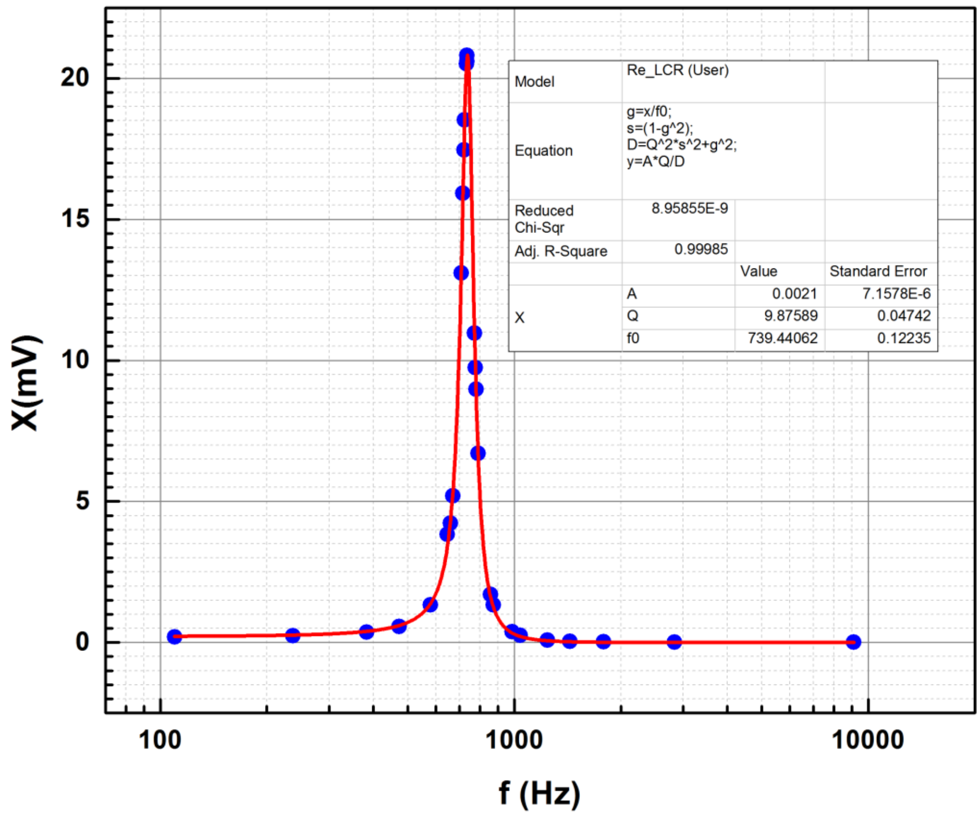

Fig2. The results of fitting of real part of the $\breve{V}_{out}(\omega)$, the equations (7)-(10) are derived for transfer function **H** components and fitting results depicted in this figure were done by using the corresponding lock-in readings, so the scaling coefficient in hare is $\frac{R}{R_0} Q \times V_{in}$

The fitting function used:

g=x/f0;
s=(1-g^2);
D=Q^2*s^2+g^2;
y=A*Q/D

Here: **g** is the reduced frequency $\gamma = \frac{\omega}{\omega_0} = \frac{f}{f_0}$ and x is measuring frequency (in Hz)

2. Imaginary part (Y) analysis

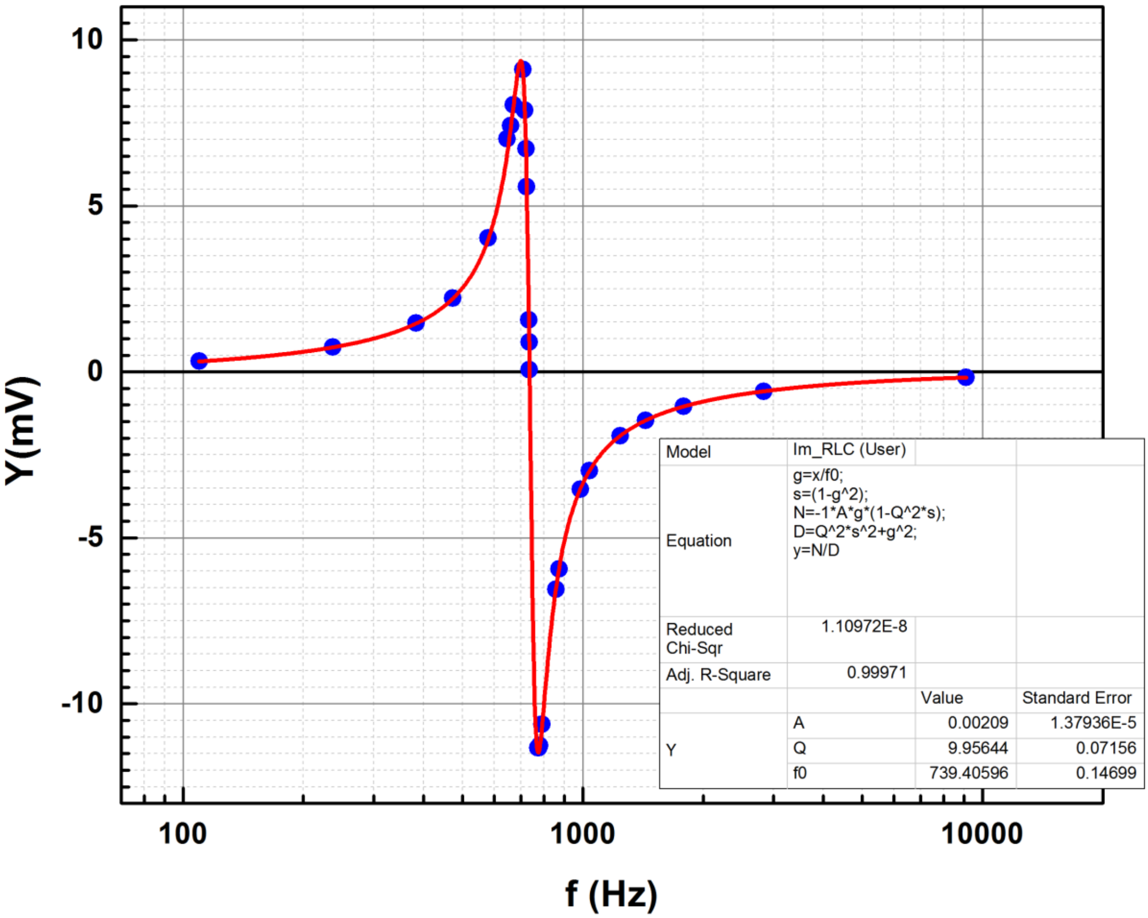

Fig3. Fitting of the imaginary part of the $\check{V}_{out}(\omega)$

The fitting function used:

g=x/f0;
s=(1-g^2);
N=-1*A*g*(1-Q^2*s);
D=Q^2*s^2+g^2;
y=N/D

3. Modulus (R)

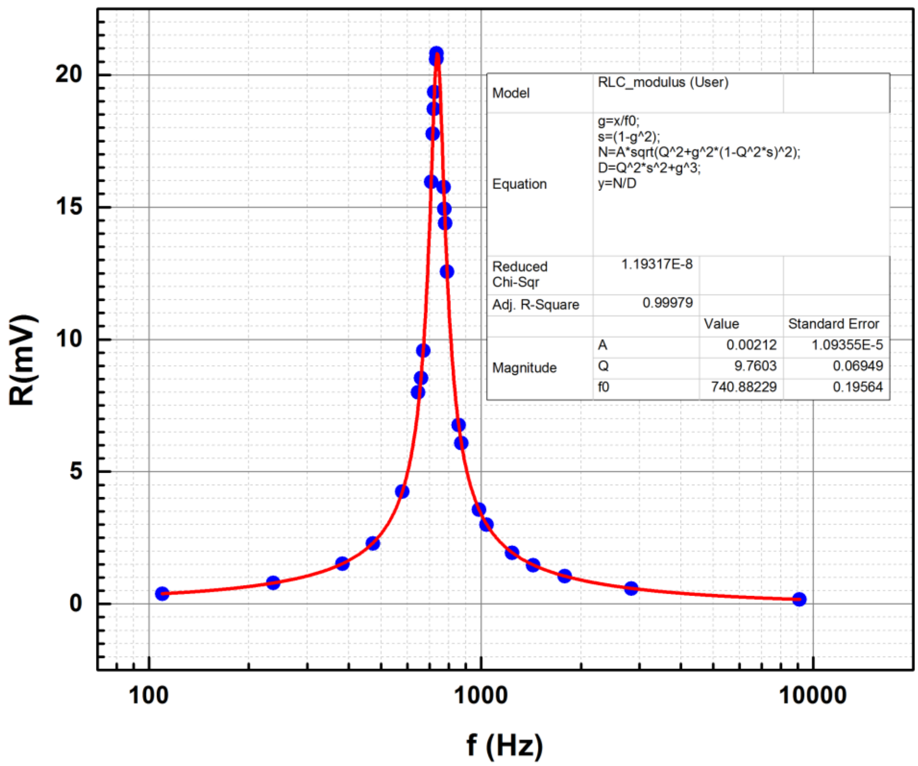

Fig4. The results of fitting of modulus $\left|\breve{V}_{out}\right| = \sqrt{X^2 + Y^2}$

The fitting function used:

g=x/f0;
s=(1-g^2);
N=A*sqrt(Q^2+g^2*(1-Q^2*s)^2);
D=Q^2*s^2+g^3;
y=N/D

4. Phase shift (θ)

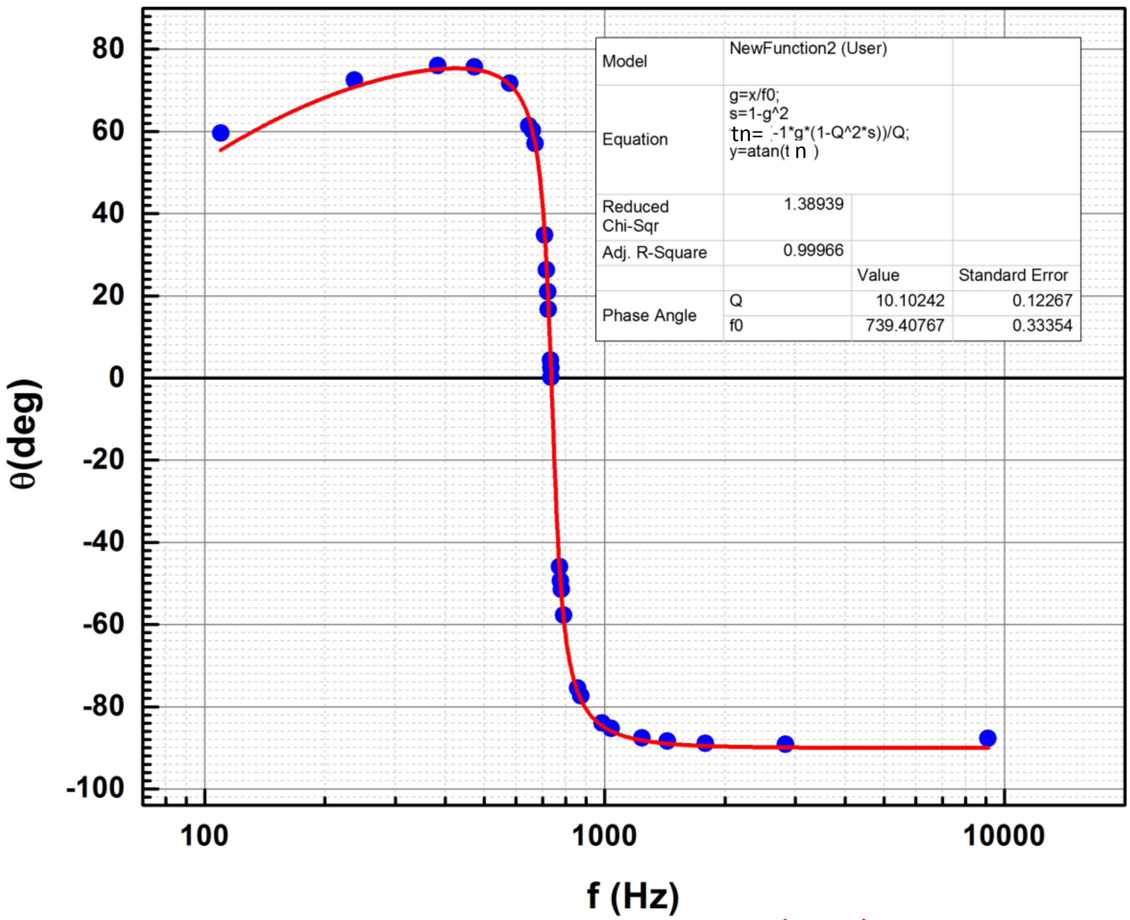

Fig5. The results of fitting of the phase shift $\theta = \arctan\left(\frac{H_{IM}}{H_{RE}}\right)$

The fitting function used:

g=x/f0;
s=1-g^2
tn=(-1*g*(1-Q^2*s))/Q;
y=atan(tn)

Comments on fitting using OriginPro

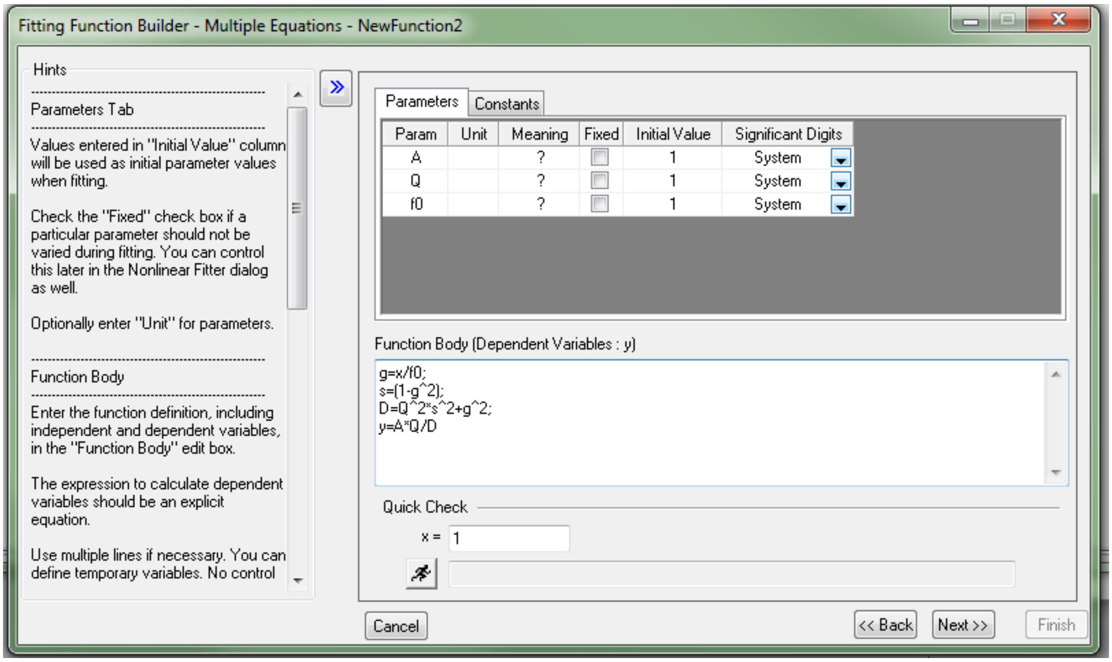

Fig.7. Fitting Function Builder. The example for the X- component fitting procedure

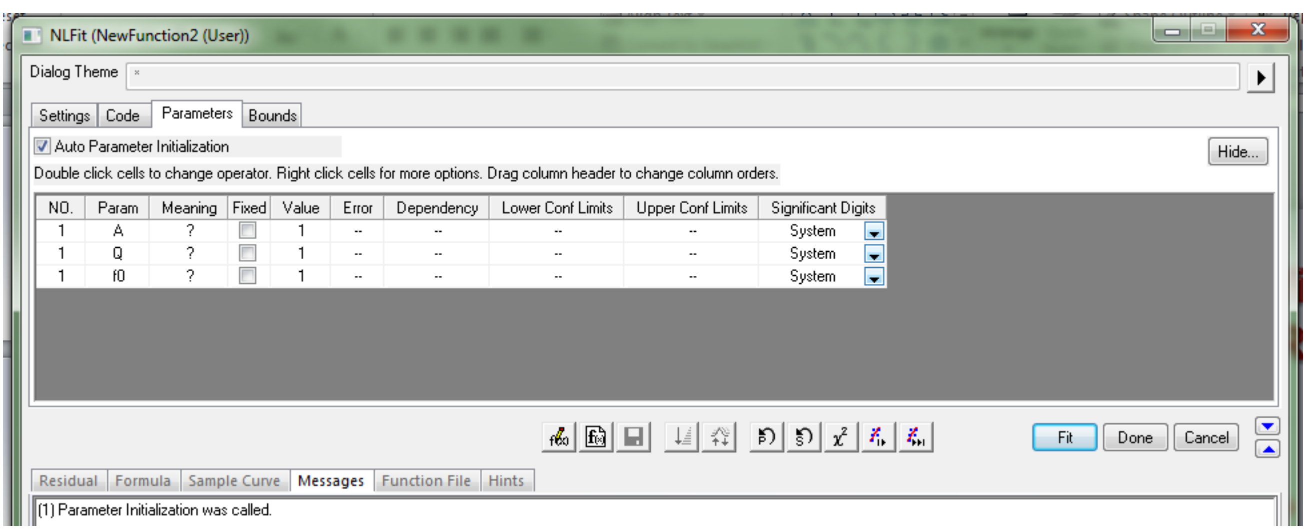

Fig.8. Fitting Parameters Window. To get successful results in fitting you have to find the realistic initial parameters numbers and plug them in Value column. In case of RLC resonance circuit **A** should be the estimation value for X component of the lock-in reading in low frequency limit, **f0** – is a rough estimation of the resonance frequency and for the quality factor **Q** the good choice is 1-20 dependable on R value but the fitting procedure is not extremely sensitive to the choice of this parameter